The paired comparison method for latent variables
An Application of the Bradley Terry Model

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2nd Workshop on Psychometric Computing
February 25th - February 26th 2010
LMU, Department for Statistics

Introduction

Instrument

Freizeit-Interessen-Test (FIT), Stangl 1991
based on Holland's (1997) RIASEC-model ....

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CONVENTIONAL: detail-oriented, organizing

Example

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Play as a musician (A)	○	○	be a conductor (E) in a folk group.

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Build a greenhouse (R)	○	○	grow and maintain rare plants (I) in your own garden.
Play as a musician (A)	○	○	be a conductor (E) in a folk group.
Produce (R)	○	○	sale (E) christmas decoration.

Standard Procedure

Sum the selected items of each scale

Compare the means of the sub-scales

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Compare the means of the sub-scales

Each item has a different attractivity

The selection of an item depends on the offered alternative

Comparison of sub-scale means is not appropriate

Way out

Use of methods for Paired Comparisons

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FIT:

60 different items

10 different items for each sub-scale

30 comparisons

e.g. $R_1 : I_1$, $I_2 : A_1$,

Analysing FIT with Paired Comparisons Methods

Problem

	[R1]	[I1]	[I2]	[A1]	[A2]	[S1]	...
[R1]	1	-1	0	0	0	0	...
[I1]	-1	1	0	0	0	0	
[I2]	0	0	1	-1	0	0	
[A1]	0	0	-1	1	0	0	
[A2]	0	0	0	0	1	-1	
[S1]	0	0	0	0	-1	1	
...	...						

Linear dependencies in the design matrix

Object Covariate

Each sub-scale is treated as an object: R I A S E C

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linear reparameterization

$$\lambda_j^O = x_{j1} \cdot R + x_{j2} \cdot I + x_{j3} \cdot A + x_{j4} \cdot S + x_{j5} \cdot E + x_{j6} \cdot C$$

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$$\lambda_{R1}^O = 1 \cdot R$$

Reparameterization Matrix

	R	I	A	S	E	C
R1	1	0	0	0	0	0
I1	0	1	0	0	0	0
I2	0	1	0	0	0	0
A1	0	0	1	0	0	0
A2	0	0	1	0	0	0
S1	0	0	0	1	0	0
⋮						

Including Subject Covariates

account for subject covariates: sex (G)

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$$\ln m_{(jk)j|g} = \mu_{(jk)jg} + \lambda^{O}_{j} - \lambda^{O}_{k} + \lambda^{OS}_{jg} - \lambda^{OS}_{kg}$$

Including Subject Covariates

account for subject covariates: sex (G)

$$\ln m_{(jk)j|g} = \mu_{(jk)jg} + \lambda^{O}_{j} - \lambda^{O}_{k} + \lambda^{OS}_{jg} - \lambda^{OS}_{kg}$$

Sample

Students of the University of Klagenfurt

G \ F	Cultural sciences	Technical sciences	total
female	45	23	68
male	5	20	25
	50	43	93

Solution

Design matrix

	y	μ	R	I	A	S	E	C	G	F
R_1	y_1	1	1	-1	0	0	0	0	1	1
I_1	y_2	1	-1	1	0	0	0	0	1	1
I_2	y_3	2	0	1	-1	0	0	0	1	1
A_1	y_4	2	0	-1	1	0	0	0	1	1
A_2	y_5	3	0	0	1	-1	0	0	1	1
$\vdots$										

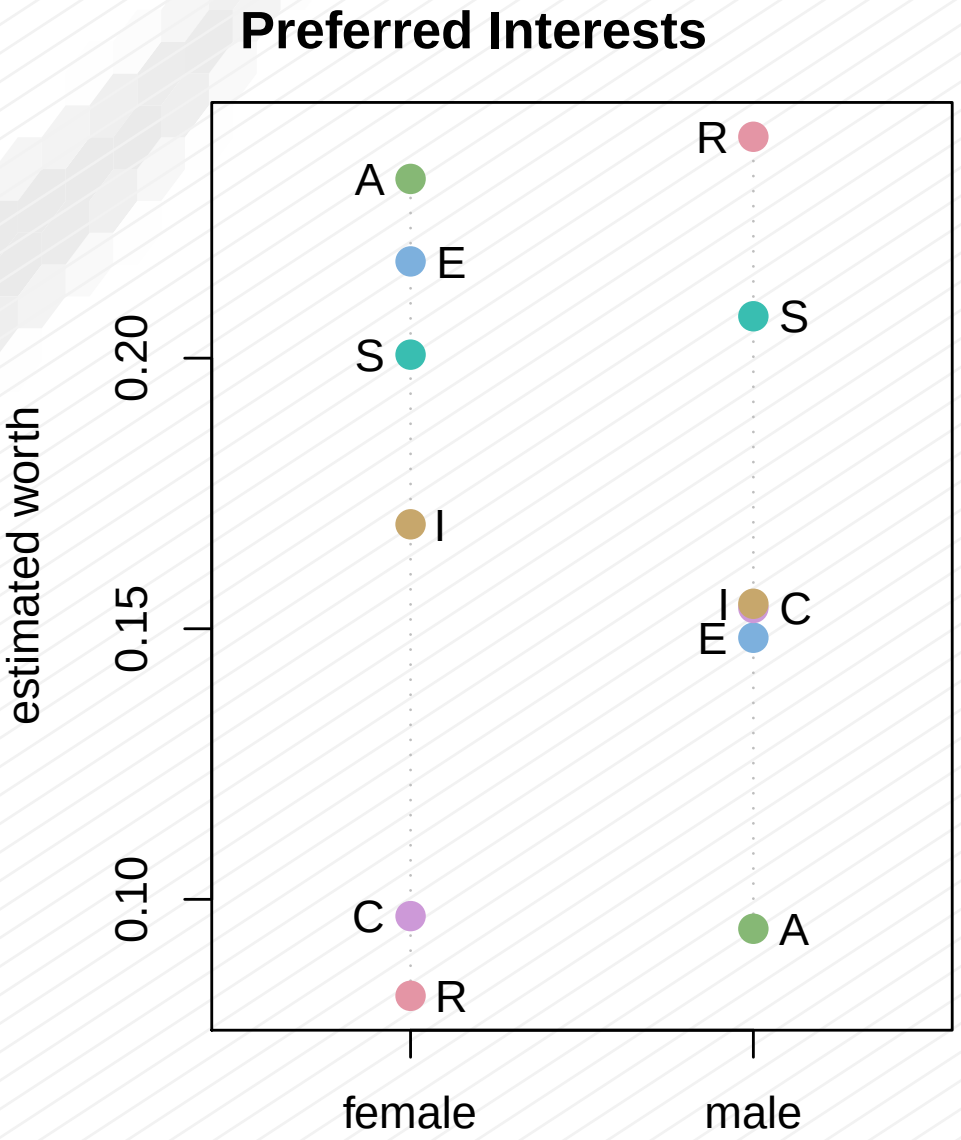
Results 1

Model Selection


```
anova(G+F,G,F)
```

	Resid. Df	Resid. Dev	Df	Deviance
G+F	165	834.58		
G	170	840.32	-5	-5.74
F	170	981.37	0	-141.05

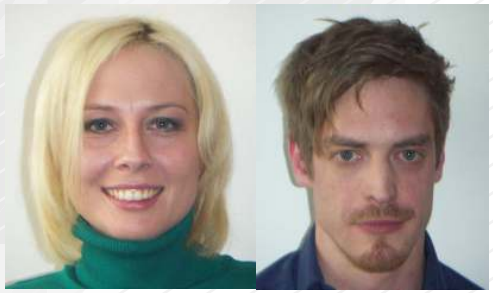
Worthplot



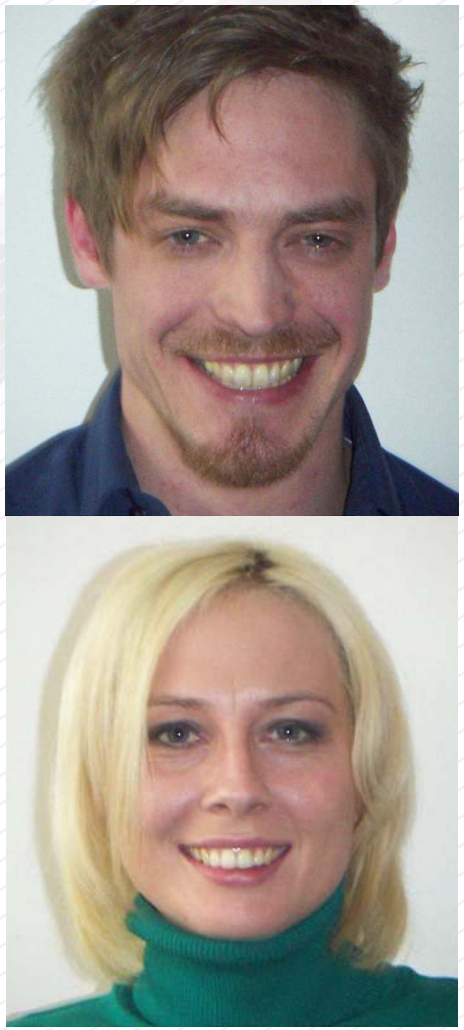
Possible Refinement

Account for the Differences

Whom would you choose as your statistical consultant?



Account for the Differences



easy items
low weights

difficult items
high weights

Item Difficulties

Where do we get these weights from?

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item difficulties from Rasch Model

Reparameterization Matrix

item difficulties are continous object covariates

choosing item R_1 wants an certain amount of the attribute 'Realistic'

	[R]	[I]	[A]	[S]	[E]	[C]
R1	0.78	0.00	0.00	0.00	0.00	0.00
I1	0.00	2.37	0.00	0.00	0.00	0.00
I2	0.00	1.36	0.00	0.00	0.00	0.00
A1	0.00	0.00	3.54	0.00	0.00	0.00
A2	0.00	0.00	0.20	0.00	0.00	0.00
S1	0.00	0.00	0.00	5.57	0.00	0.00

Design Matrix

	y	μ	R	I	A	S	E	C	G	F
R_1	y_1	1	0.78	-2.37	0	0	0	0	1	1
I_1	y_2	1	-0.78	2.37	0	0	0	0	1	1
I_2	y_3	2	0	1.36	-3.54	0	0	0	1	1
A_1	y_4	2	0	-1.36	3.54	0	0	0	1	1
A_2	y_5	3	0	0	0.20	-5.57	0	0	1	1
$\vdots$										

Account for Item Difficulties

consider different item difficulties

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consider subject covariates sex and faculty

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$$\ln m_{(jk)j|l} = \mu_{(jk)jl} + \lambda_j^O - \lambda_k^O + \lambda_{jl}^{OS} - \lambda_{kl}^{OS}$$

$$\lambda_j^O = x_{j1} \cdot R + x_{j2} \cdot I + x_{j3} \cdot A + x_{j4} \cdot S + x_{j5} \cdot E + x_{j6} \cdot C$$

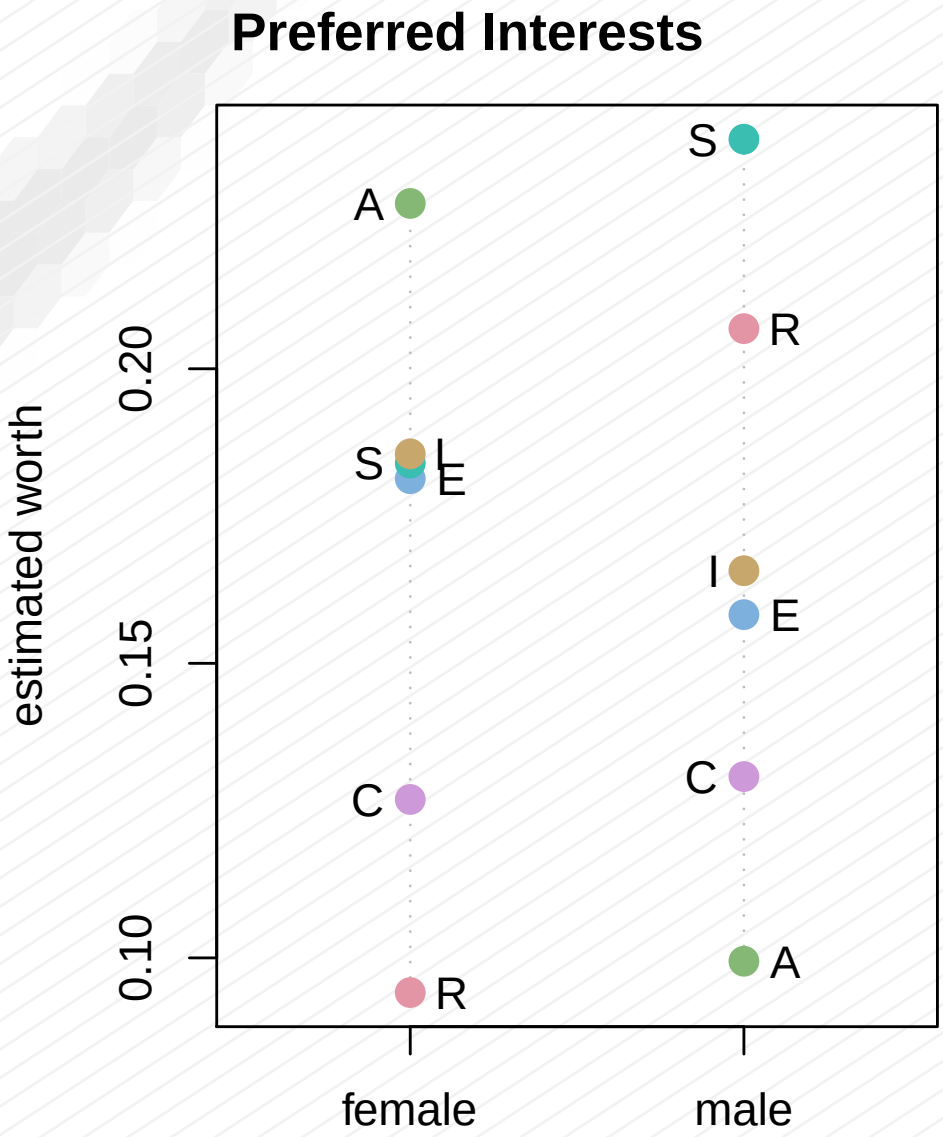
Results 2

Model Selection


```
anova(G+F,G,F)
```

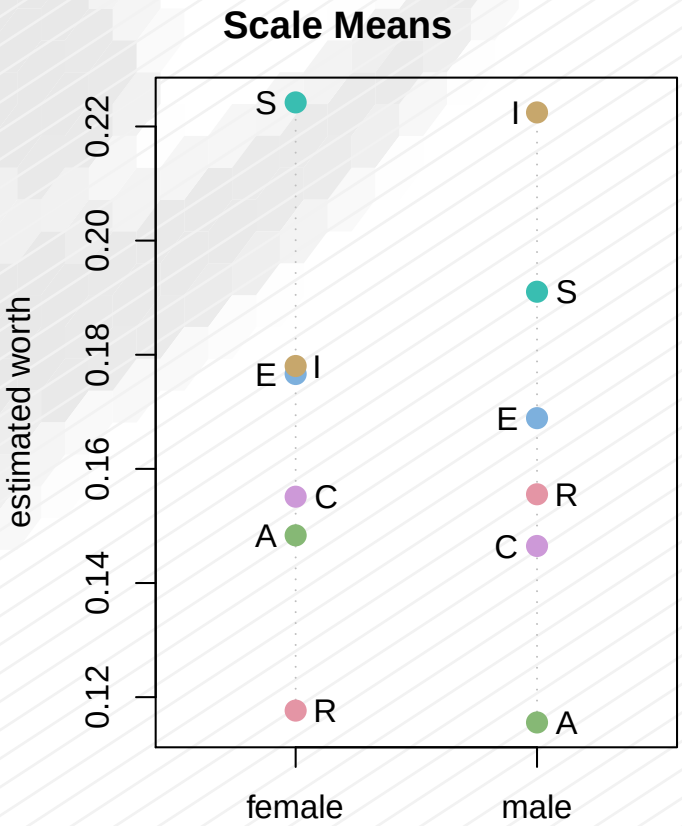
	Resid. Df	Resid. Dev	Df	Deviance
G+F	162	725.91		
G	168	754.81	-6	-28.90
F	168	938.71	0	-183.90

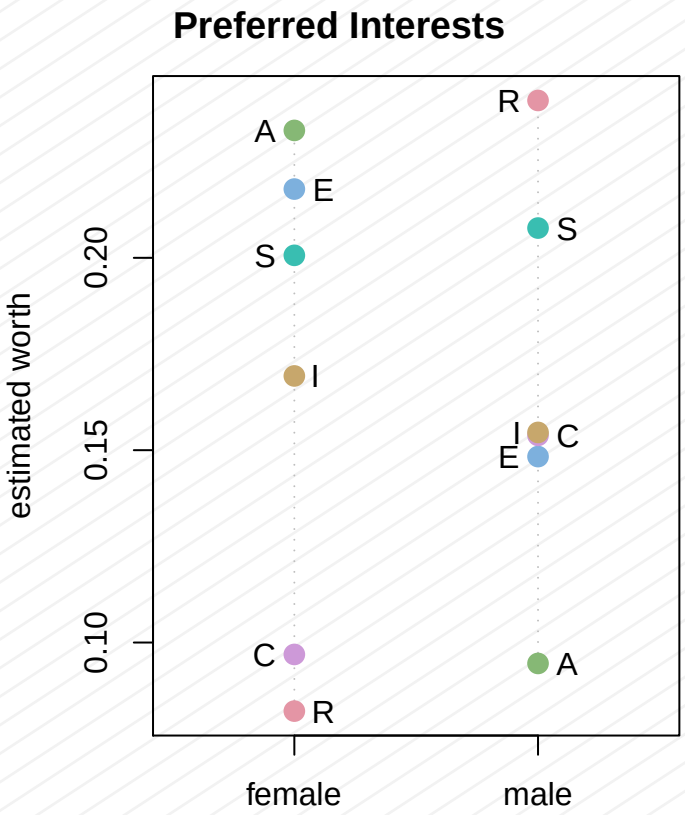
Worthplot



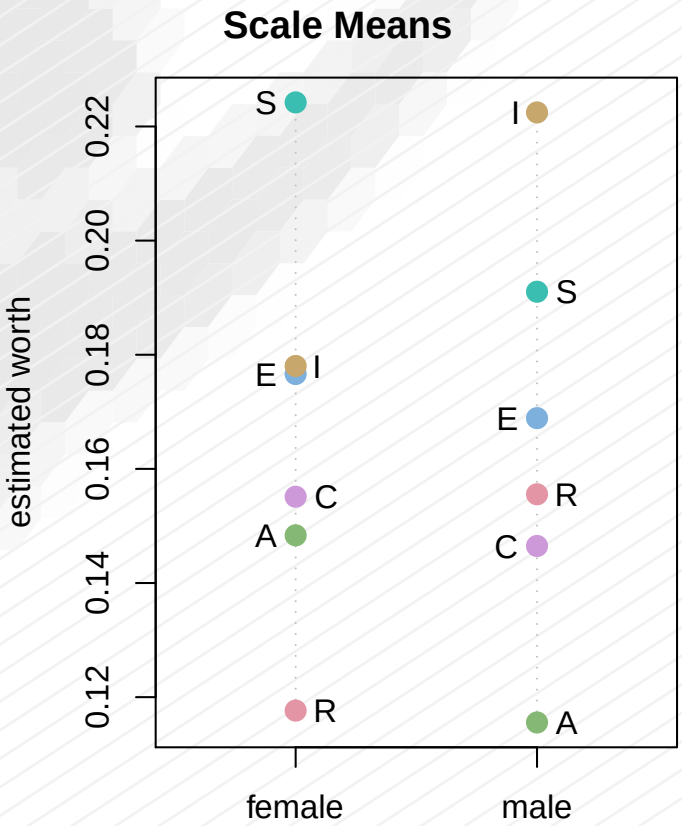
Comparison of Results

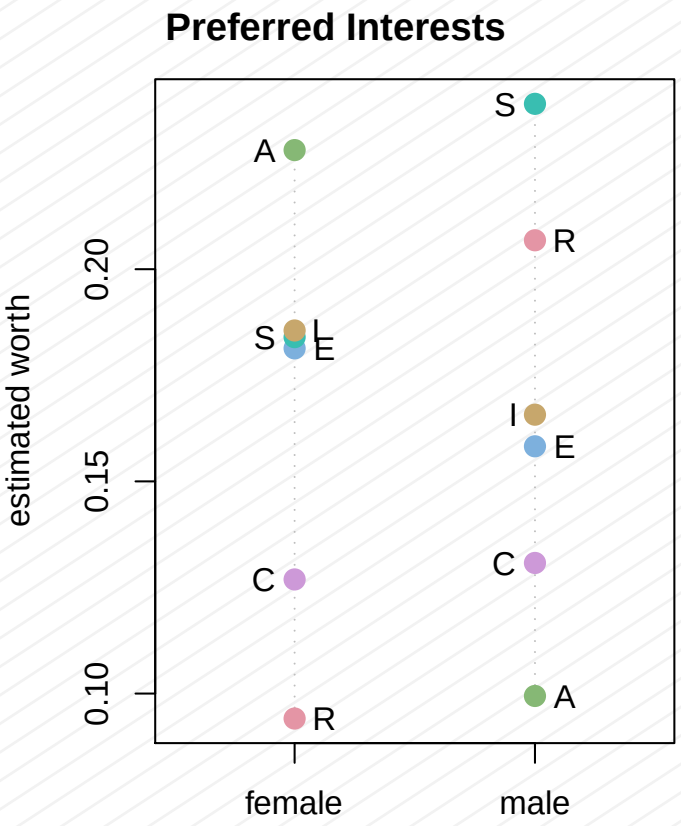
Sub-Scale Means vs. Categorical Object Covariate



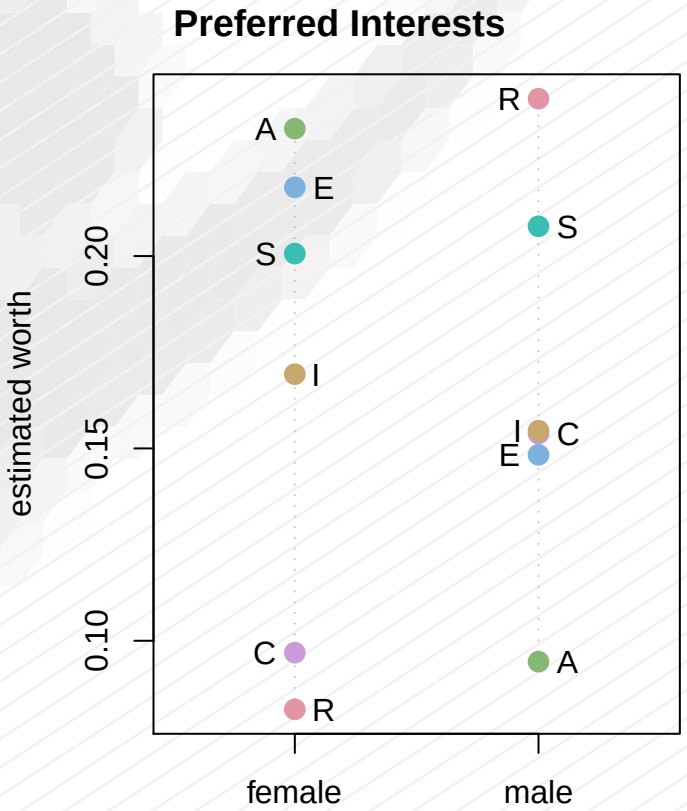


Sub-Scale Means vs. Continous Object Covariate

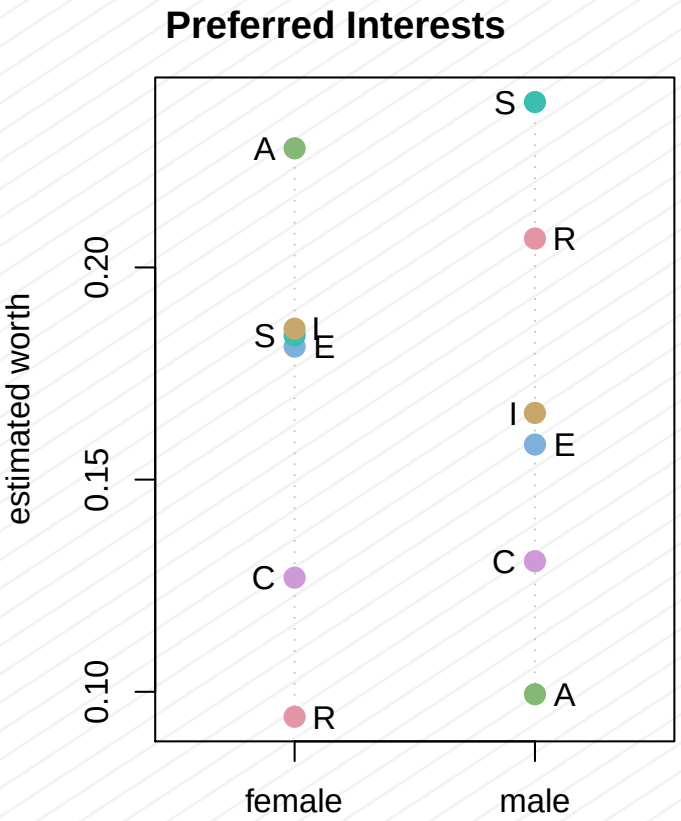




Categorical Object Covariates vs. Continous Object Covariates



AIC: 1909.7



AIC: 1828.2

Conclusio

the method of analysis makes a difference

further refinement is still needed

Thank you

Thank you!

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