The paired comparison method for latent variables An Application of the Bradley Terry Model

> Almut Thomas Michaela Gareiß Regina Dittrich Reinhold Hatzinger

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O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Introduction

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

REALISTIC: practical, physical

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

REALISTIC: practical, physical

INVESTIGATIVE: intellectual, scientific

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

REALISTIC: practical, physical

INVESTIGATIVE: intellectual, scientific

ARTISTIC: creative, independent

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

REALISTIC: practical, physical

INVESTIGATIVE: intellectual, scientific

ARTISTIC: creative, independent

SOCIAL: supporting, helping

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

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ARTISTIC: creative, independent

SOCIAL: supporting, helping

ENTERPRISING: competitive, persuading

Introduction

O Instrument

O ExampleO Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Freizeit-Interessen-Test (FIT), Stangl 1991 based on Holland's (1997) RIASEC-model

REALISTIC: practical, physical

INVESTIGATIVE: intellectual, scientific

ARTISTIC: creative, independent

SOCIAL: supporting, helping

ENTERPRISING: competitive, persuading

CONVENTIONAL: detail-oriented, organizing

Introduction

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Which of the two alternatives would you prefer?

Introduction	
--------------	--

O Instrument

O Example

O Standard ProcedureO Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Which of the two alternatives would you prefer?

Build a greenhouse (R) o o

grow and maintain rare plants (I) in your own garden.

Introduction	
O Instrument	
O Example	
O Standard Procedure	
O Way out	TATI:
Analysing FIT with Paired	VV NIC
Comparisons Methods	
Results 1	B
Possible Refinement	
Results 2	
Comparison of Results	

Which of the two alternatives would you prefer?

Build a greenhouse (R) $\circ \circ$ grow and maintain

grow and maintain rare plants (I) in your own garden.

Play as a musician (A) $\circ \circ$ be a conductor (E) in a folk group.

Introduction	
O Instrument	
O Example	
O Standard Procedure	
O Way out	
Analysing FIT with Paired	Which of the two alterna
Comparisons Methods	
<u>Companio methodo</u>	
Results 1	Build a greenhouse (R)
Possible Refinement	Dunia a greennouse (it)
Results 2	
Comparison of Results	
	Play as a musician (A)
	Droduco (D)
	Produce (K)

alternatives would you prefer?

ouse (R)

• • grow and maintain rare plants (I) in your own garden.

 \circ \circ be a conductor (E) in a folk group.

sale (E) 0 0 christmas decoration.

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Sum the selected items of each scale

Compare the means of the sub-scales

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Sum the selected items of each scale

Compare the means of the sub-scales

Each item has a different attractivity

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Sum the selected items of each scale

Compare the means of the sub-scales

Each item has a different attractivity

The selection of an item depends on the offered alternative

Comparison of sub-scale means is not appropriate

Way out

Introduction

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Use of methods for Paired Comparisons

Way out

Introduction

O Instrument

O Example

O Standard Procedure

O Way out

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

Use of methods for Paired Comparisons

60 different items

FIT:

10 different items for each sub-scale

30 comparisons

e.g. $R_1 : I_1, I_2 : A_1,$

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object

Covariates

O Reparameterization

Matrix

O Including Subject

Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

Analysing FIT with Paired Comparisons Methods

Problem

Introduction								
Analysing FIT with Paired Comparisons Methods								
O Problem								
O Object Covariate		[0.1	TTA	17.1	TA 1	[]]	10.1	
O Categorical Object		[K1]		[12]		[A2]	[51]	•••
O Reparameterization	[R1]	1	-1	0	0	0	0	
Matrix O Including Subject	[I1]	-1	1	0	0	0	0	
Covariates	[12]	0	0		-1	0	0	
O Sample								
O Solution	[A1]	0	0	-1	1	0	0	
Results 1	[A2]	0	0	0	0	1	-1	
Possible Refinement	[S1]	0	0	0	0	-1	1	
Results 2								
	· / · · · /	/						

Comparison of Results

Linear dependencies in the design matrix

Object Covariate

Introduction	
Analysing FIT with Paired Comparisons Methods	
O Problem	
O Object Covariate	
 O Categorical Object Covariates O Reparameterization Matrix O Including Subject Covariates O Sample 	
O Solution	
Results 1	Each sub-scale is treated as an object: R I
Possible Refinement	
Results 2	
Comparison of Results	

ASEC

Object Covariate

Introduction	
Analysing FIT with Paired Comparisons Methods	
O Problem	
O Object Covariate	
 O Categorical Object Covariates O Reparameterization Matrix O Including Subject Covariates 	
O Sample	
O Solution	
Results 1	Each sub-scale is treated as an object: R I A S E C
Possible Refinement	
Results 2	Each item is assigned to a sub-scale
Comparison of Results	

Object Covariate

Introduction	
Analysing FIT with Paired Comparisons Methods	
O Problem	
O Object Covariate	
 O Categorical Object Covariates O Reparameterization Matrix O Including Subject Covariates 	
O Sample	
O Solution	
Results 1	Each sub-scale is treated as an object: R I A S E C
Possible Refinement	
Results 2	Each item is assigned to a sub-scale
Comparison of Results	

Introduction

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object Covariates

O Reparameterization Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

The fact that an item belongs to one scale is treated as categorical object covariate

Introduction

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object Covariates

O Reparameterization Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

The fact that an item belongs to one scale is treated

as categorical object covariate

e.g. item R_1 has the attribute Realistic \Rightarrow 1 on the object covariate R

Introduction

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object Covariates

O Reparameterization Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

The fact that an item belongs to one scale is treated as categorical object covariate

e.g. item R_1 has the attribute Realistic $\Rightarrow 1$ on the object covariate R

$$\ln m_{(jk)j} = \mu_{(jk)j} + \lambda_j^O - \lambda_k^O$$

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object

Covariates

O ReparameterizationMatrixO Including Subject

Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

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$$\ln m_{(jk)j} = \mu_{(jk)j} + \lambda_j^O - \lambda_k^O$$

linear reparameterization

 $\lambda_{j}^{O} = x_{j_{1}} \cdot R + x_{j_{2}} \cdot I + x_{j_{3}} \cdot A + x_{j_{4}} \cdot S + x_{j_{5}} \cdot E + x_{j_{6}} \cdot C$

Introduction

Analysing FIT with Paired Comparisons Methods O Problem O Object Covariate O Categorical Object Covariates O Reparameterization Matrix O Including Subject

Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

The fact that an item belongs to one scale is treated as categorical object covariate

e.g. item R_1 has the attribute Realistic \Rightarrow 1 on the object covariate R

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linear reparameterization

$$\lambda_j^O = x_{j_1} \cdot R + x_{j_2} \cdot I + x_{j_3} \cdot A + x_{j_4} \cdot S + x_{j_5} \cdot E + x_{j_6} \cdot C$$
$$\lambda_{R_1}^O = \mathbf{1} \cdot R$$

Introduction							
Analysing FIT with Paired Comparisons Methods							
O Problem							
O Object Covariate							
O Categorical Object Covariates							
O Reparameterization Matrix		R	Ι	A	S	Е	C
O Including Subject Covariates	R1	1	0	0	0	0	0
O Sample	- L1	~	1	0			
O Solution		0		0	0	0	0
Results 1	I2	0	1	0	0	0	0
Possible Refinement	A1	0	0	1	0	0	0
Results 2	A2	0	0	1	0	0	0
Comparison of Results	S1	0	0	0	1	0	0

Including Subject Covariates

Introduction

Analysing FIT with Paired Comparisons Methods

O Problem

O Object Covariate

O Categorical Object

Covariates

O Reparameterization

Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

account for subject covariates: sex (G)

Including Subject Covariates

Introduction

Analysing FIT with Paired Comparisons Methods O Problem

OProblem

O Object Covariate

O Categorical Object

Covariates

O Reparameterization

Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

account for subject covariates: sex (G)

$$\ln m_{(jk)j|g} = \mu_{(jk)jg} + \lambda_j^O - \lambda_k^O + \lambda_{jg}^{OS} - \lambda_{kg}^{OS}$$

Including Subject Covariates

Introduction

Analysing FIT with Paired Comparisons Methods O Problem

OProblem

O Object Covariate

O Categorical Object

Covariates

O Reparameterization

Matrix

O Including Subject Covariates

O Sample

O Solution

Results 1

Possible Refinement

Results 2

Comparison of Results

account for subject covariates: sex (G)

$$\ln m_{(jk)j|g} = \mu_{(jk)jg} + \lambda_j^O - \lambda_k^O + \lambda_{jg}^{OS} - \lambda_{kg}^{OS}$$

Sample

Introduction

Analysing FIT with Paired Comparisons Methods	0.1
O Problem	Students
O Object Covariate	
O Categorical Object	
O Reparameterization	
O Including Subject	
O Sample	
O Solution	
Results 1	
Possible Refinement	
Results 2	
Comparison of Results	

of the University of Klagenfurt

F G	Cultural sciences	Technical sciences	total
female male	45 5	23 20	68 25
	50	43	93

Solution

Introduction

Analysing FIT with Paired Design matrix **Comparisons Methods O** Problem O Object Covariate y μ **O** Categorical Object Covariates **O** Reparameterization R_1 y_1 1 Matrix **O** Including Subject I_1 y_2 1 Covariates O Sample I_2 y_3 2 **O** Solution Results 1 A_1 y_4 2 Possible Refinement A_2 y_5 3 Results 2 Comparison of Results

R I A S E CG F 1 -1 0 0 0 0 1 1 -1 1 0 0 0 0 1 1 0 1 -1 0 0 1 1 0 0 -1 1 0 0 0 1 1 1 -1 0 1 1 0 0 0

Analysing FIT with Paired Comparisons Methods

Results 1

O Model Selection

O Worthplot

Possible Refinement

Results 2

Comparison of Results

Results 1

Model Selection

Introduction					
Analysing FIT with Paired Comparisons Methods					
Results 1					
O Model Selection					
O Worthplot					
Possible Refinement	anova(G	+F,G,F)			
Results 2					
Comparison of Results	Resi	d. Df R	esid. Dev	Df	Deviance
	G+F	165	834.58		
	G	170	840.32	-5	-5.74
	F	170	981.37	0	-141.05

Worthplot



Preferred Interests



Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

O Account for the Differences O Account for the Differences O Item Difficulties

O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

Possible Refinement

Account for the Differences

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences

O Account for the Differences

O Item Difficulties O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results













Whom would you choose as your statistical consultant?

Thomas Gareiß Dittrich & Hatzinger

Account for the Differences

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the

Differences

O Item Difficulties O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results





easy items low weights





difficult items high weights

Item Difficulties

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the Differences

O Item Difficulties

O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

Where do we get these weights from?

Item Difficulties

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the Differences

O Item Difficulties

O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

Where do we get these weights from?

item difficulties from Rasch Model

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the Differences O Item Difficulties O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

item difficulties are continous object covariates

choosing item R_1 wants an certain amount of the attribute 'Realistic'

	[R]	[1]	[A]	[S]	[E]	[C]
R 1	0.78	0.00	0.00	0.00	0.00	0.00
I1	0.00	2.37	0.00	0.00	0.00	0.00
I2	0.00	1.36	0.00	0.00	0.00	0.00
A1	0.00	0.00	3.54	0.00	0.00	0.00
A2	0.00	0.00	0.20	0.00	0.00	0.00
S1	0.00	0.00	0.00	5.57	0.00	0.00

Design Matrix

Introduction											
Analysing FIT with Paired Comparisons Methods Results 1		y	μ	R	Ι	A	S	Е	С	G	F
Possible Refinement O Account for the Differences O Account for the	R_1 I_1	y ₁ y ₂	1 1	0.78 -0.78	-2.37 2.37	0 0	0 0	0 0	0 0	1 1	1 1
Differences O Item Difficulties O Reparameterization Matrix	I_2 A_1	y_3 y_4	2 2	0	1.36 -1.36	-3.54 3.54	0 0	0 0	0 0	1 1	1 1
O Design Matrix O Account for Item Difficulties	A ₂	<i>y</i> ₅	3	0	0	0.20	-5.57	0	0	1	1
Results 2 Comparison of Results											

Account for Item Difficulties

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the Differences O Item Difficulties O Reparameterization Matrix O Design Matrix O Account for Item Difficulties Results 2

Comparison of Results

consider different item difficulties

Account for Item Difficulties

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the Differences O Account for the Differences O Item Difficulties

O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

consider different item difficulties

consider subject covariates sex and faculty

Account for Item Difficulties

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement O Account for the

O Account for t

Differences

O Account for the

Differences

O Item Difficulties O Reparameterization Matrix

O Design Matrix

O Account for Item Difficulties

Results 2

Comparison of Results

consider different item difficulties

consider subject covariates sex and faculty

$$\ln m_{(jk)j|l} = \mu_{(jk)jl} + \lambda_j^O - \lambda_k^O + \lambda_{jl}^{OS} - \lambda_{kl}^{OS}$$

$$\lambda_{j}^{O} = x_{j_{1}} \cdot R + x_{j_{2}} \cdot I + x_{j_{3}} \cdot A + x_{j_{4}} \cdot S + x_{j_{5}} \cdot E + x_{j_{6}} \cdot C$$

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

O Model Selection O Worthplot

Comparison of Results

Results 2

Model Selection

Introduction	
Analysing FIT with Paired Comparisons Methods	
Results 1	
Possible Refinement	
Results 2	an
O Model Selection	

Ο

O Worthplot

Comparison of Results

nova(G+F,G,F)

Res	id. Df R	esid. Dev	Df	Deviance
G+F	162	725.91		
G	168	754.81	-6	-28.90
F	168	938.71	0	- 183.90

Worthplot



Preferred Interests



Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results

O Sub-Scale Means vs.
Categorical Object Covariate
O Sub-Scale Means vs.
Continous Object Covariate
O Categorical Object
Covariates vs. Continous
Object Covariates
O Conclusio
O Thank you

Comparison of Results

Sub-Scale Means vs. Categorical Object Covariate



Sub-Scale Means vs. Continous Object Covariate



Categorical Object Covariates vs. Continous Object Covariates



AIC: 1909.7

AIC: 1828.2

Conclusio

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results O Sub-Scale Means vs. Categorical Object Covariate O Sub-Scale Means vs. Continous Object Covariate O Categorical Object Covariates vs. Continous Object Covariates

O Conclusio

O Thank you

the method of analysis makes a difference further refinement is still needed

Thank you

Introduction

Analysing FIT with Paired Comparisons Methods

Results 1

Possible Refinement

Results 2

Comparison of Results O Sub-Scale Means vs. Categorical Object Covariate O Sub-Scale Means vs. Continous Object Covariate O Categorical Object Covariates vs. Continous Object Covariates O Conclusio O Thank you

THANK YOU!

Almut.Thomas@uni-klu.ac.at