Model-based recursive partitioning for Bradley-Terry models

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Goal of model based partitioning

Motivation

- The preference scaling of a population of subjects may not be homogeneous.
- Different groups of subjects with certain characteristics may show different preference scalings.
- For each group, a separate Bradley-Terry (BT) model with different parameters might hold.
- The groups may be unknown a priori.

Goal

Identify groups of subjects with homogeneous model parameters.

Steps of BT model partitioning algorithm

1. Fit a BT model to the paired comparisons of all subjects in the current (sub-)sample, starting with the full sample.
2. Assess the stability of the BT model parameters with respect to each available covariate.
3. If there is significant instability, split the sample along the covariate with the strongest instability and use the cutpoint with the highest improvement of the model fit.
4. Repeat steps 1–3 recursively in the resulting subsamples until there are no more significant instabilities (or the subsample is too small).

Fitting the Bradley-Terry model

In a paired-comparison, the probabilities of choosing the first alternative (1), the second alternative (2), or of being undecided (3) are (Davidson, 1970)

$$
\begin{aligned}
p_{jj'1} &= \frac{\pi_j}{\pi_j + \pi_{j'} + \nu\sqrt{\pi_j \pi_{j'}}} \\
p_{jj'2} &= \frac{\pi_{j'}}{\pi_j + \pi_{j'} + \nu\sqrt{\pi_j \pi_{j'}}} \\
p_{jj'3} &= \frac{\nu\sqrt{\pi_j \pi_{j'}}}{\pi_j + \pi_{j'} + \nu\sqrt{\pi_j \pi_{j'}}}
\end{aligned}
$$

With $\theta = (\log(\pi_1), \ldots, \log(\pi_{k-1}), \log(\nu))^\top$, the model may be fitted using an auxiliary log-linear model (or a logit model, when there are no ties).

Attractiveness of Germany's Next Topmodels 2007

Method

- $N = 192$ stratified by gender and age, 48 in each subgroup
- Presented with photographs of the top six contestants
- Each participant did all $6 \cdot 5/2 = 15$ pairwise comparisons

Research question

Does perceived attractiveness of the contestants vary with gender and age, and with previous knowledge of the participants?

q1 Do you recognize the women on the pictures?/Do you know the TV show Germany's Next Topmodel?

q2 Did you watch Germany's Next Topmodel regularly?

q3 Did you watch the final show of Germany's Next Topmodel?/Do you know who won Germany's Next Topmodel?

The top six contestants

Barbara

Anni

Hana

Fiona

Mandy

Anja

Binary paired-comparison judgments

Which of these two women do you find more attractive?

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The paircomp class

paircomp is designed for holding paired comparisons of k objects measured for n subjects.

```
Topmodel2007$pref[1:5]
[1] {Brb > Ann, Brb > Han, Ann > Han, Brb > Fin, Ann < Fin...}
[2] {Brb < Ann, Brb < Han, Ann < Han, Brb < Fin, Ann > Fin...}
[3] {Brb < Ann, Brb < Han, Ann < Han, Brb < Fin, Ann < Fin...}
[4] {Brb < Ann, Brb > Han, Ann > Han, Brb > Fin, Ann > Fin...}
[5] {Brb < Ann, Brb < Han, Ann < Han, Brb < Fin, Ann > Fin...}
```

Under the hood:

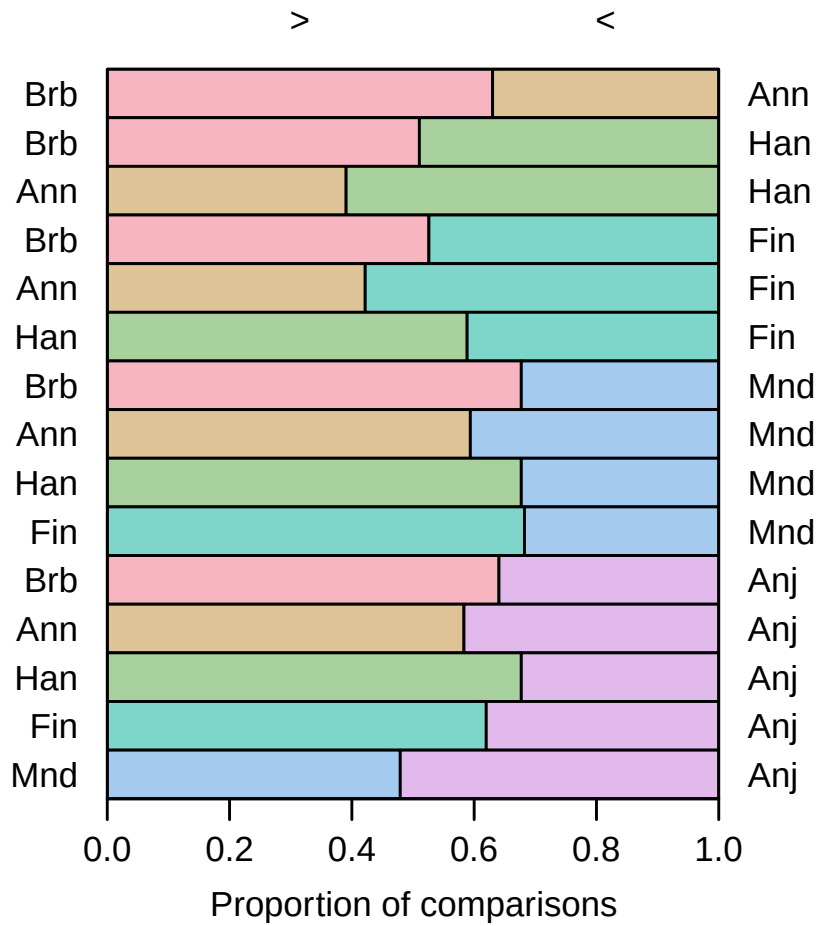
```
> unclass(Topmodel2007$pref[1:2])
     1:2 1:3 2:3 1:4 2:4 3:4 1:5 2:5 3:5 4:5 1:6 2:6 3:6 4:6 5:6
[1,]   1   1   1   1  -1  -1   1  -1  -1   1  -1  -1  -1  -1  -1
[2,]  -1  -1  -1  -1   1   1   1   1   1   1   1   1   1   1   1
attr(,"labels")
[1] "Barbara" "Anni"    "Hana"    "Fiona"   "Mandy"   "Anja"
attr(,"mscale")
[1] -1  1
attr(,"ordered")
[1] FALSE
```

Descriptive statistics

Aggregate judgments, $N = 192$ per pair

```
summary(Topmodel2007$pref)
                     >   <
Barbara : Anni      121  71
Barbara : Hana       98  94
Anni    : Hana       75 117
Barbara : Fiona     101  91
Anni    : Fiona      81 111
Hana    : Fiona     113  79
Barbara : Mandy     130  62
Anni    : Mandy     114  78
Hana    : Mandy     130  62
Fiona   : Mandy     131  61
Barbara : Anja      123  69
Anni    : Anja      112  80
Hana    : Anja      130  62
Fiona   : Anja      119  73
Mandy   : Anja       92 100
```

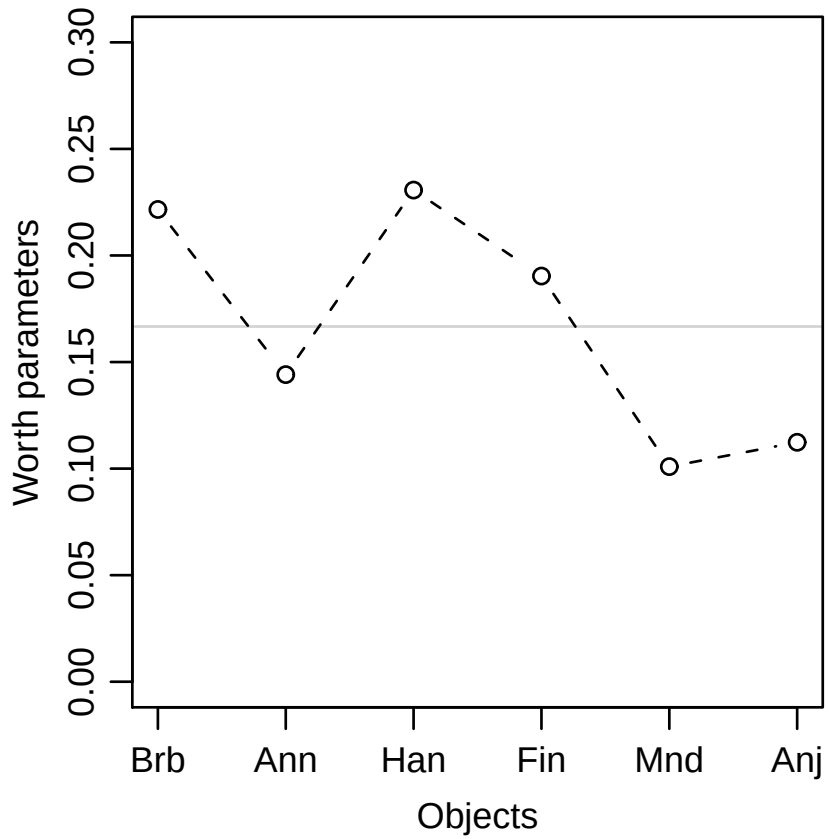
```
plot(Topmodel2007$pref)
```



Bradley-Terry model for the entire sample

```
tm <- btReg.fit(Topmodel2007$pref)  # workhorse function

worth(tm)                                  # worth parameters
Barbara    Anni    Hana   Fiona   Mandy    Anja
   0.22    0.14    0.23    0.19    0.10    0.11
plot(tm)
```



Partitioning the Bradley-Terry model

```
tmt <- bttree(preference ~ ., data=Topmodel2007, minsplit=5)
```

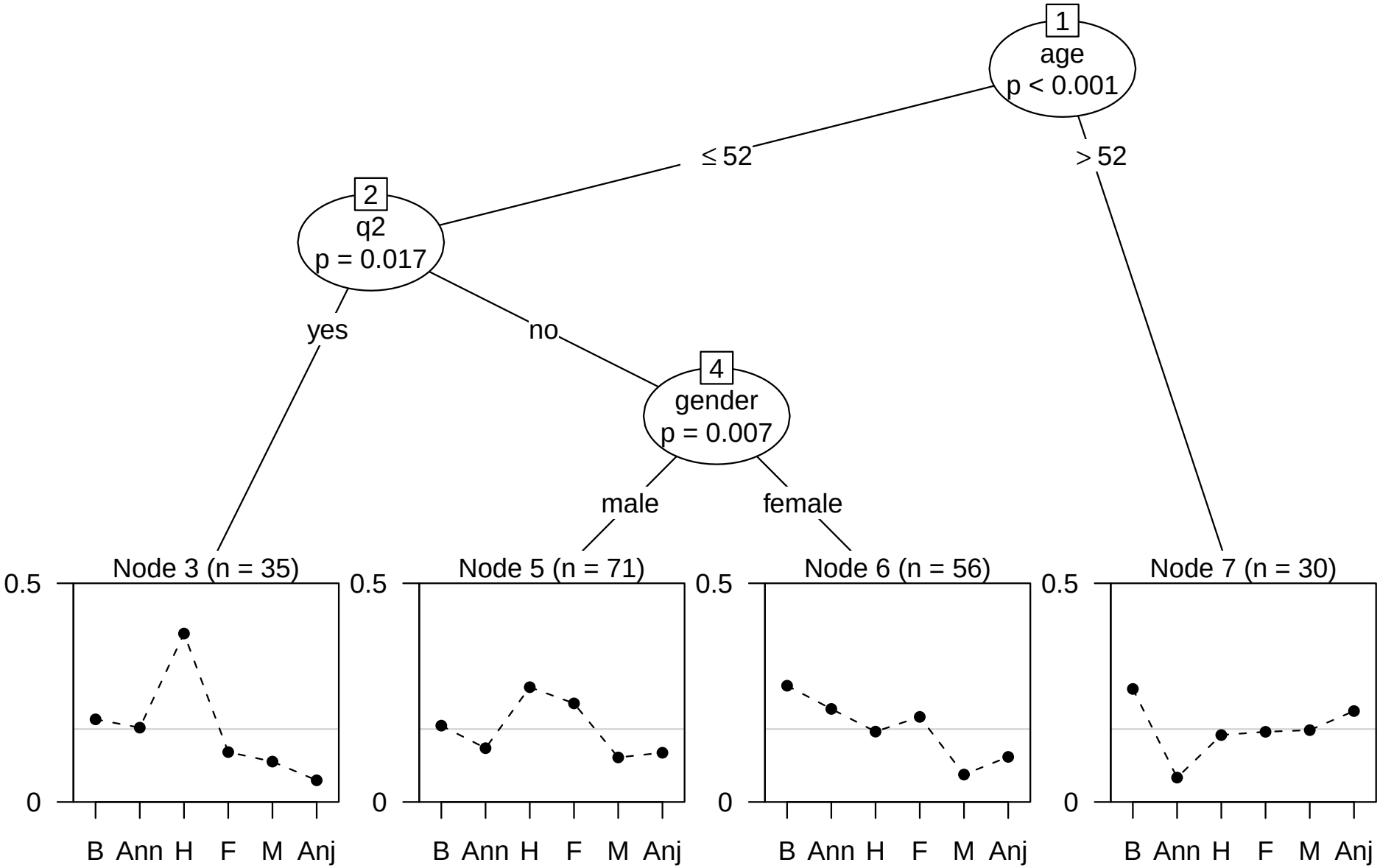
Test for structural change

```
sctest(tmt, node=1)
```

	gender	age	q1	q2	q3
statistic	17.088	32.357	12.632	19.839	6.759
p.value	0.022	0.001	0.128	0.007	0.745

Use age for splitting the sample, and fit model in the subsamples. Continue recursively.

Partitioned Bradley-Terry model



Conclusions

With model based recursive partitioning you can

- find groups of subjects with similar model parameters
- by means of partitioning the covariate space.

The advantages of this approach are that

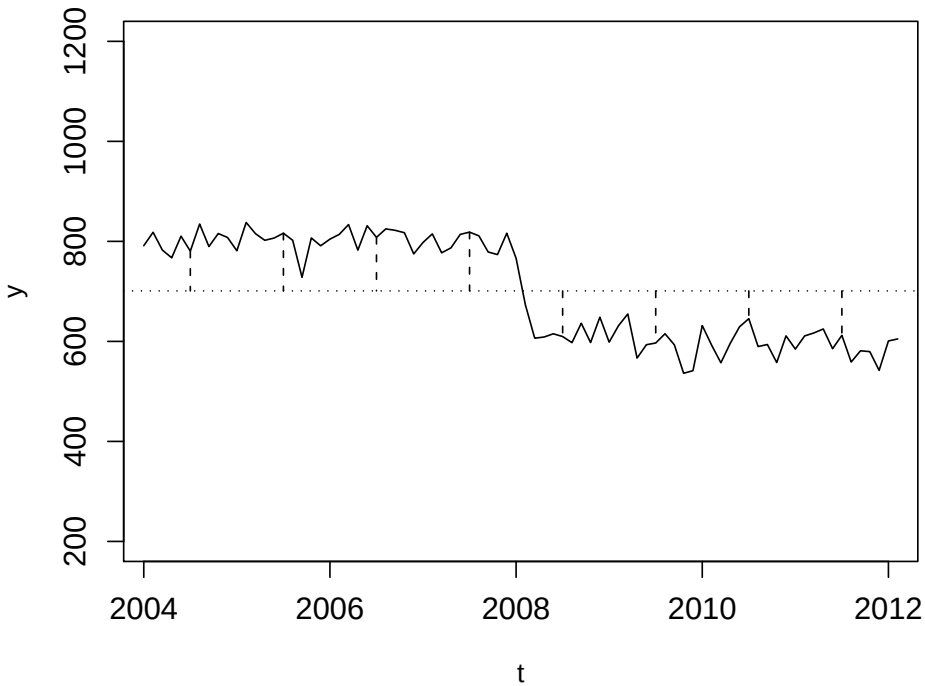
- the groups need not be known
- combinations of relevant covariates are identified
- interactions between covariates are incorporated
- continuous covariates are discretized in an optimal, data-driven way for splitting

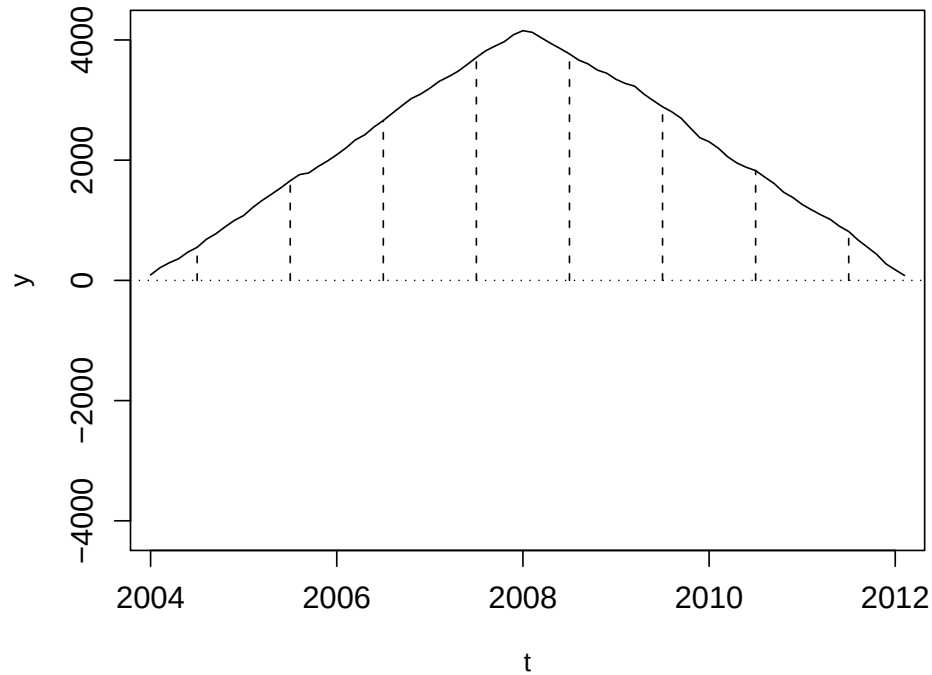
Thank you for your attention

http://CRAN.r-project.org/package=psychotree

Strobl, C., Wickelmaier, F., & Zeileis, A. (in press). Accounting for individual differences in Bradley-Terry models by means of recursive partitioning. *Journal of Educational and Behavioral Statistics*.

Structural change





Structural change