Trees and Rasch-Trees

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LMU

CEST
Center for Empirical Studies

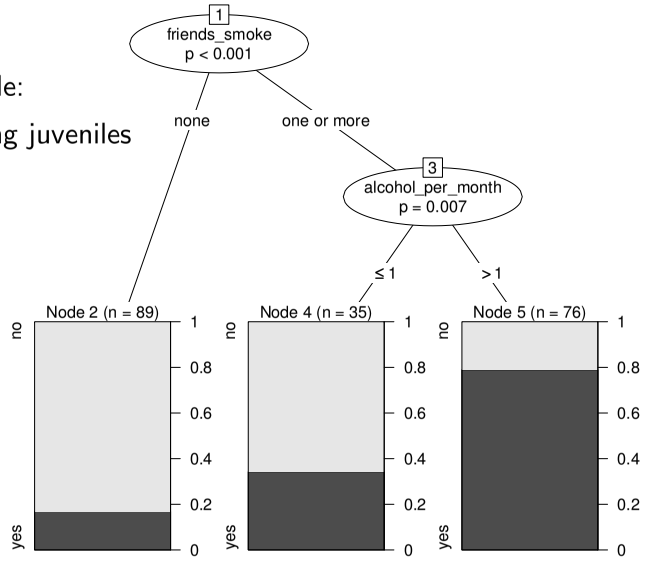
Outline

Trees and Rasch-Trees
Carolin Strobl, Julia Kopf, Achim Zeileis
CART
MOB
Testing for DIF
Graphical test
Latent classes
Model-based partitioning
Example
Conclusion
Literatur

Classification and regression trees (CART)

Example:

Smoking juveniles



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Optimal cutpoint selection

Example: $\chi^2 - test$

operationalization 1:

X binary, Y binary

		Y	
		1	2
X	1	h_{11}	h_{12}
	2	h_{21}	h_{22}

$$\sum_i \sum_j \frac{\left(h_{ij} - \tilde{h}_{ij}\right)^2}{\tilde{h}_{ij}} \sim \chi^2(1)$$

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Example: $\chi^2 - test$

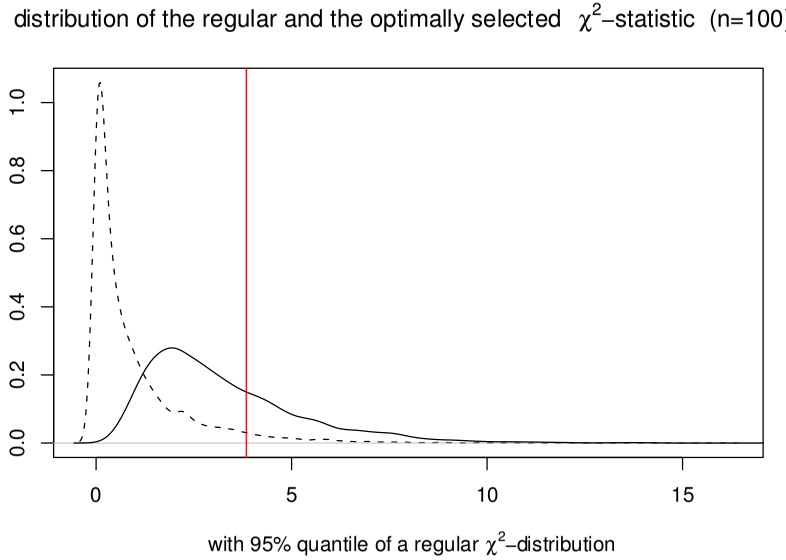
operationalization 2:

X continuous, Y binary

		Y	
		1	2
X	$x \leq c$	h_{11}	h_{12}
	$x > c$	h_{21}	h_{22}

$\sum_i \sum_j \frac{(h_{ij} - \tilde{h}_{ij})^2}{\tilde{h}_{ij}} \sim \chi^2(1)$???

Optimal cutpoint selection



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Variable selection bias

combination of optimal cutpoint and variable selection leads to bias in the standard algorithms:

variables with many categories and continuous variables are systematically preferred (cf., e.g., White & Liu, 1994; Loh & Shih, 1997; Jensen & Cohen, 2000; Kim & Loh, 2001; Dobra & Gehrke, 2001; Strobl, Boulesteix & Augustin, 2007)

this bias affects the R-packages `tree` (Ripley, 2007) and `rpart` (Therneau & Atkinson, 2006), but not function `ctree` in package `party` (Hothorn, Hornik, Strobl & Zeileis, 2011)

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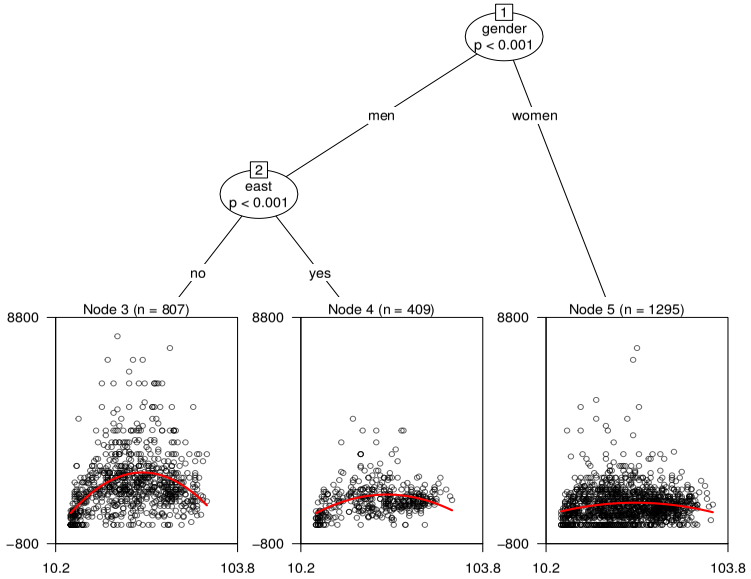
Model-based recursive partitioning (MOB)



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Model-based recursive partitioning (MOB)

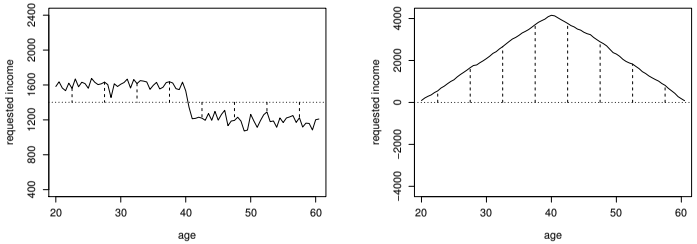
identify groups of people with different parameters by means of tests for parameter instability:

- individual contributions to the score-funktion

$$\psi(y_i, \theta) = \frac{\partial \Psi(y_i, \theta)}{\partial \theta}$$

- cumulated over all values of covariate ℓ

$$W_\ell(t) = \hat{V}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor n \cdot t \rfloor} \psi(y_{(i|\ell)}, \hat{\theta})$$



- under H_0 the path fluctuates randomly around zero ($\rightarrow$ Brownian bridge; Zeileis & Hornik, 2007)

Model-based recursive partitioning (MOB)

optimally selected Lagrange-Multiplier test-statistics

- for continuous covariates

$$S_\ell = \max_{i=\underline{i},\ldots,\overline{i}} \left(\frac{i}{n} \cdot \frac{n-i}{n}\right)^{-1} \left\| W_\ell\left(\frac{i}{n}\right) \right\|_2^2$$

- and similarly for categorical covariates

with known distributions

$\Rightarrow$ p-values = split selection criteria

function `mob` in package `party` (Hothorn et al., 2011)

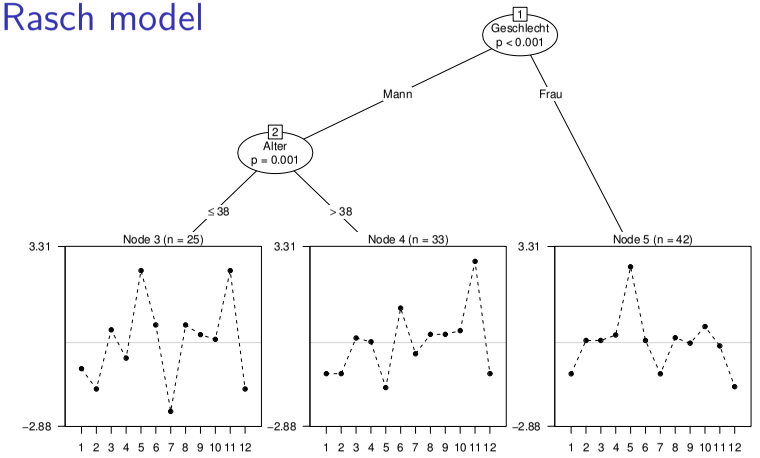
MOB versus CART

Berk (Berk, 2006, p. 236) about algorithmic methods such as classification and regression trees:

> *"With algorithmic methods, there is no statistical model in the usual sense; no effort has been made to represent how the data were generated. And no apologies are offered for the absence of a model. There is a practical data analysis problem to solve that is attacked directly with procedures designed specifically for that purpose."*

Model-based recursive partitioning combines theory-based and data-driven approach

New: Model-based recursive partitioning for the Rasch model



Strobl, Wickelmaier & Zeileis (2010); Strobl, Kopf & Zeileis (2010)

R-package `psychotree` (Zeileis, Strobl & Wickelmaier)

Differential Item Functioning (DIF)

is present when one or more items of a test

- are easier or harder to solve for certain subjects
- even though they have the same latent trait

Standard model tests

- tests for k given groups
  graphical test, Andersen's Likelihood-Ratio Test, Wald Tests
  - \+ straightforward interpretation
  - – only detect DIF in specified groups
- latent-class approach
  Rost's "Mixed" Rasch model
  - \+ identifies previously unknown groups with DIF
  - – groups are not directly interpretable
  - ⇒ 2nd step: describe groups with covariates

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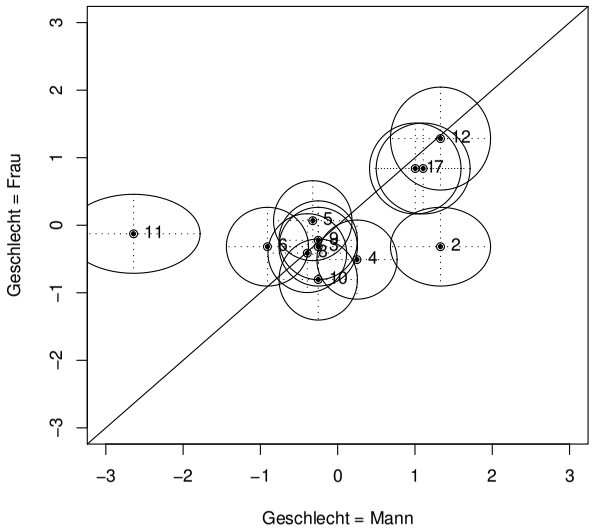
Standard model tests



R-package eRm (Mair, Hatzinger & Maier, 2010)

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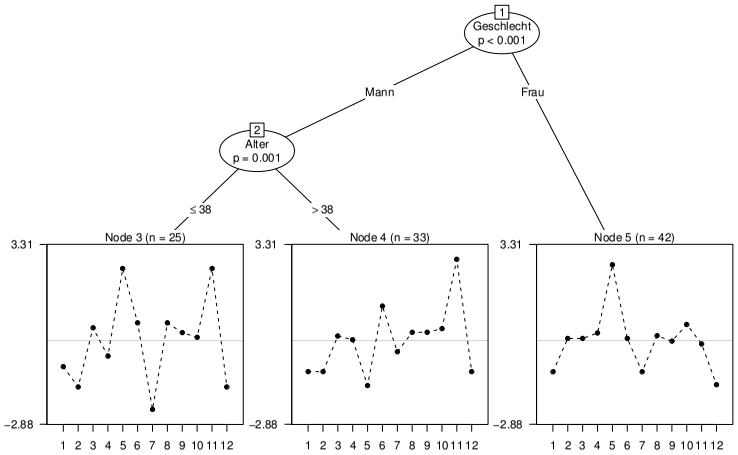
New: Model-based recursive partitioning

+ identifies previously unknown groups with DIF
+ straightforward interpretation



Example: "Students-PISA"

data from the SPIEGEL "Students-PISA" survey

- open-access online survey on general education
- each partcipant was randomly assigned one of 24 questionnaires, consisting of 45 items from 6 topics: politics, history, economics, culture and natural sciences
- questions were either multiple-choice or open
- recorded response: correct/wrong

results presented here are for one exemplary questionnaire, N = 30 188

Example: "Students-PISA"

exemplary questions:

- history: "In which century did the Thirty Years' War take place?" 17th
- economics: "Which internet-company took over the media-group Time Warner?" AOL
- culture: "Which city is the setting for the novel 'Buddenbrooks'?" Lübeck

Example: "Students-PISA"

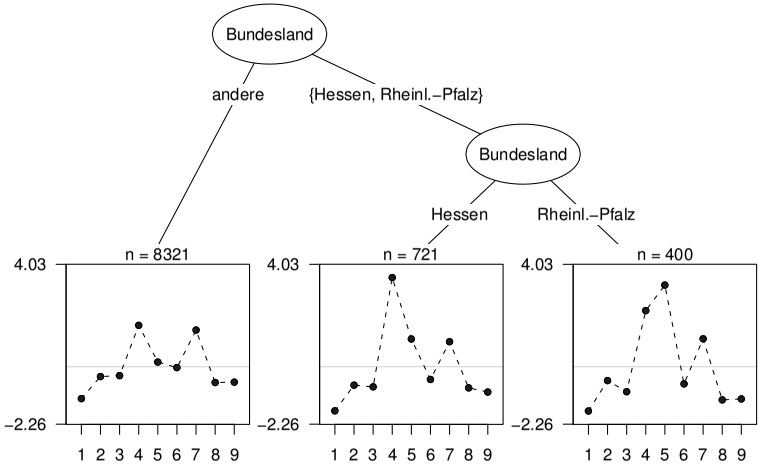
curious finding:

those participants who received their Abitur in Rheinland-Pfalz perform significantly better in the test

possible explanations:

- they are just smarter
- they have an unfair advantage ⇒ DIF

Example: "Students-PISA"



Nr. 4: Where is Hessen? (indicate location on a map)

Nr. 5: What is the capital of Rheinland-Pfalz? (Mainz)

psychological impact of DIF

- test is no longer specifically objective
- fair comparisons between the groups are impossible

⇒ eliminate DIF-items from the test

in our example:
eliminating items 4 and 5 eliminates group differences, i.e.,

the supposed group difference was only an artefact of the test construction!

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Conclusion

- DIF can lead to severe misinterpretations
- statistical tools for DIF-diagnosis
  - approaches for given groups
    - graphical test
    - Andersen's Likelihood-Ratio Test
  - Rost's latent-class approach
  - model-based recursive partitioning („Rasch-trees")
- interpretability of diagnostic tools can help understand the psychological sources of DIF
  but: observed covariates may be proxies for true causes

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