Comparing Missing Values Handling Algorithms in the Context of the Rasch Model

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Model Equation

Observations: $v = 1 \ldots n$
Items: $i = 1 \ldots k$

$$p\left(x_{vi} \middle| \theta_v, \beta_i\right) = \frac{e^{x_{vi}(\theta_v - \beta_i)}}{1 + e^{\theta_v - \beta_i}} = \frac{(\xi_v \varepsilon_i)^{x_{vi}}}{1 + \xi_v \varepsilon_i}$$

$$\xi_v = e^{\theta_v}$$

$$\varepsilon_i = e^{-\beta_i}$$

Conditional ML Estimation (CML) I

$$L_C(\boldsymbol{\varepsilon}|\mathbf{r}) = \prod_{v=1}^{n} \prod_{i=1}^{k} \varepsilon_i^{x_{vi} b_{vi}} \gamma_{r_v}^{-1}$$ with $\gamma_{r_v}(\boldsymbol{\varepsilon}, \mathbf{B}) = \sum_{\mathbf{x}_v | r_v} \prod_{i=1}^{k} (\varepsilon_i b_{vi})^{x_{vi}}$

Data matrix **X**

$$\begin{array}{ccccc} 1 & 0 & 1 & a & a \\ 0 & 1 & 0 & a & a \\ 1 & 1 & 0 & 1 & 1 \\ a & a & 1 & 1 & 0 \\ a & a & 1 & 0 & 1 \end{array}$$

Design matrix **B**

$$\begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}$$

CML II

Assumption:
The design matrix B *is assumed to be known before answers are obtained (cf. Molenaar, 1995, p. 40), e.g. when using testlets.*

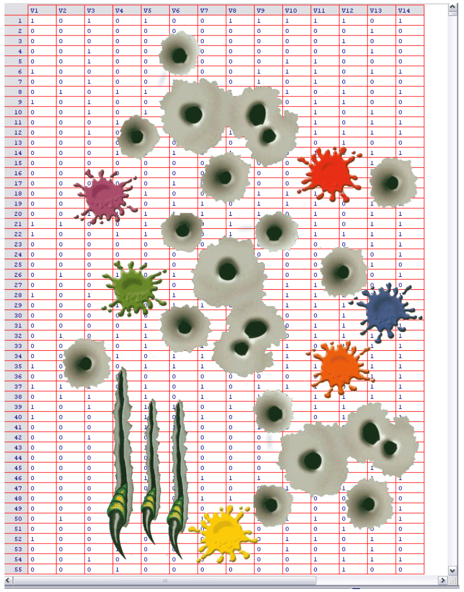
To establish a common scale for all item parameters, link items must exist (well vs. ill-conditioned data). This can be warranted by adequately assembling the testlets.

Molenaar, I. W. (1995). Estimation of Item Parameters.
In: G.H. Fischer & I.W. Molenaar (Eds.). *Rasch Models. Foundations, Recent Developments, and Applications.* (pp. 39-51). NY: Springer.

Problem

Problem:
Missing values appear in the course of testing, hence the assumption does not hold.

Question:
Is the design matrix **B** a valid means for handling missing values not known prior to data acquisition?



Method *(i)*

Simulation Study:
Data sets conforming to the Rasch Model were generated and missingness was induced according to the taxonomy of Rubin (1976) [MCAR, MAR, NMAR].

Special focus: MCAR vs. NMAR:

MCAR: a given percentage of values were deleted randomly across the data matrix
NMAR: the probability of missingness was determined according to a 4PL-like model:

$$p\left(m_{vi}\middle|\theta_v,\beta_i,a,b,c,d\right)=c+\frac{(1-c-d)}{1+e^{a(\theta_v-\beta_i-b)}}$$

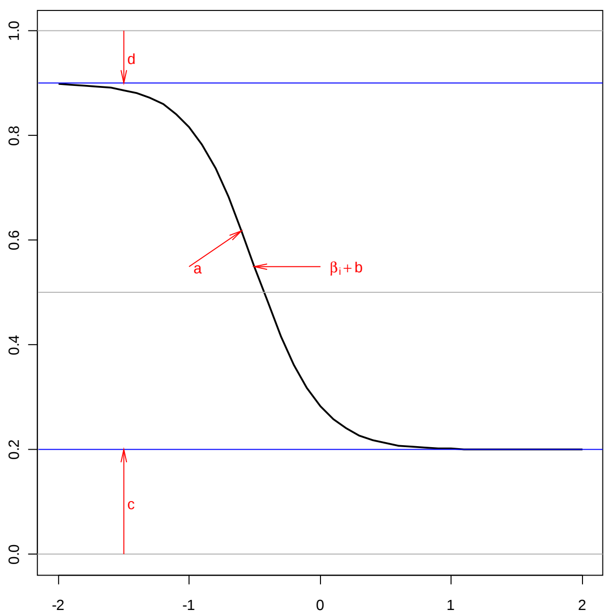
+ Intermediate step:
Use person parameter estimate as propensity to produce a missing value.

Rubin, D. B. (1976). Inference and missing data. *Biometrika, 63*, 581–592.

$$p\left(m_{vi}\middle|\theta_v,\beta_i,a,b,c,d\right)=c+\frac{(1-c-d)}{1+e^{a(\theta_v-\beta_i-b)}}$$

Method *(ii)*

p(missing|NMAR)



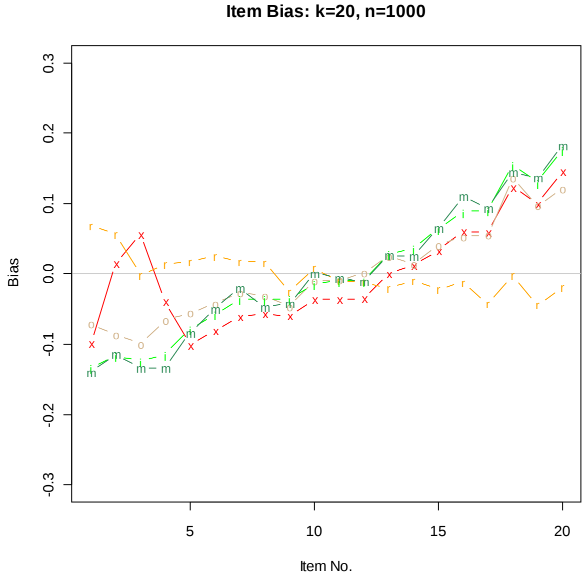
Method *(iii)*

Missing Values Handling Methods

1) Treat as structural missings, i.e. pretend, they were never presented to the testee (involving **B** with zeros inserted where missing values occured).
2) Assume no answer given = no answer known; testee preferres to omit a question to taking the risk of a wrong answer. Missings are replaced by zeros.
3) Opposite to 2: Missing values are replaced by ones (e.g. because testee did not want to admit support of nuklear power plants or a right wing party in a survey; social desirability; ...).
4) Assume testee was, say, distracted, but would have been able to sometimes respond correctly and sometimes not; however, we do not know which. Missing values are replaced by 0 or 1 drawn randomly from a Bernoulli with p=.5
5) „Mean imputation": Replace missings by draws from a Bernoulli with $p=\frac{1}{n}\sum_v x_{vi}^{[obs]}$
6) „Model based imputation": Replace missings by draws from a Bernoulli with $\hat{p}_{vi}=p\left(+\middle|\hat{\theta}_v,\hat{\beta}_i\right)$ (two step method).

Results *(i)*

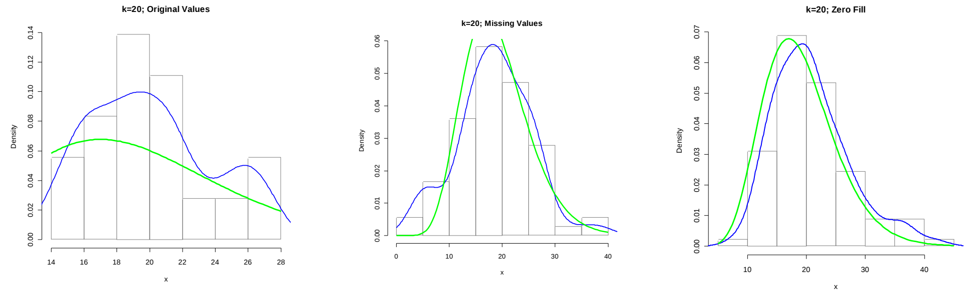
MCAR – Item Bias



increasing difficulty

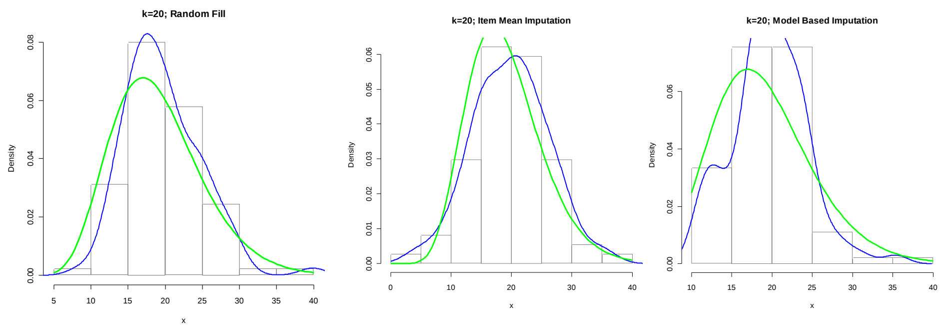
Results *(ii)*

MCAR – LR Test



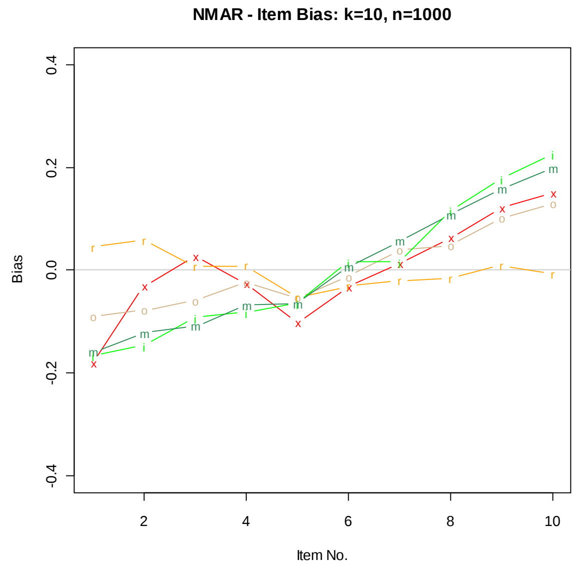
Results *(iii)*

MCAR – LR Test



Results *(iv)*

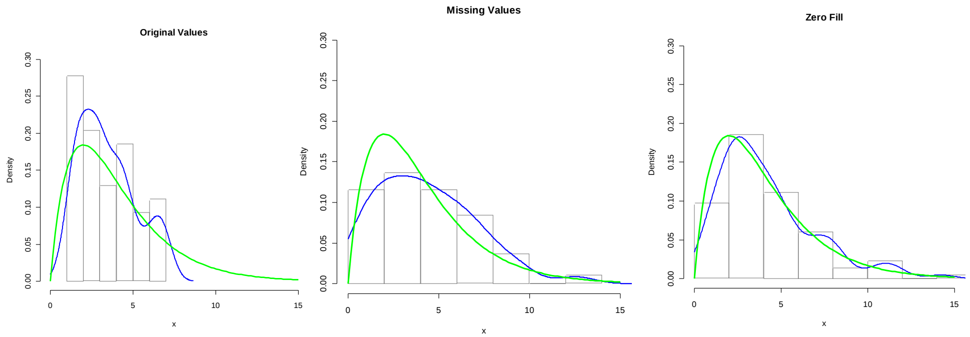
NMAR (1) – Item Bias



Results *(v)*

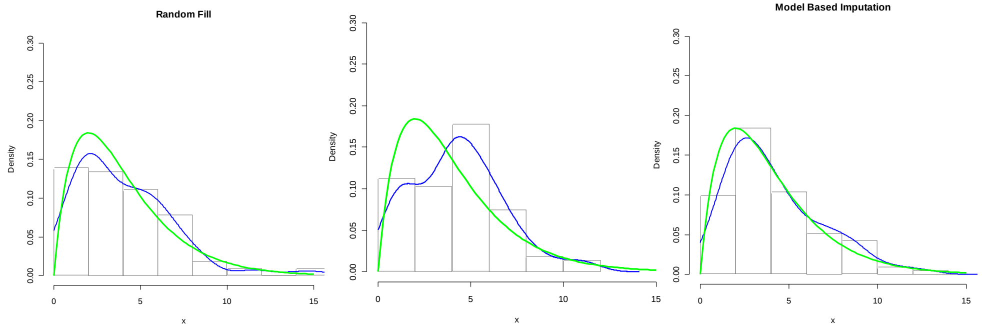
k=5

NMAR (1) – LR-Test



Results *(vi)*

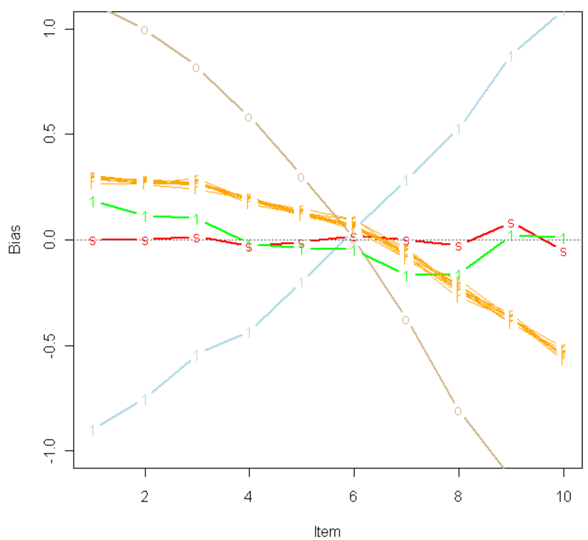
NMAR (1) – LR-Test



in fact: This principle is not NMAR!

Results *(vii)*

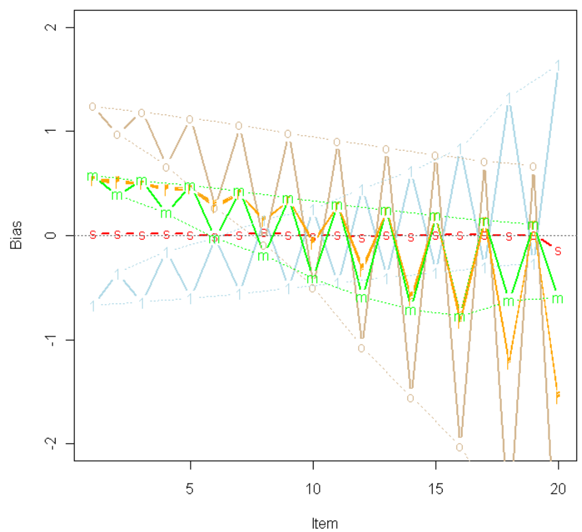
NMAR (2) – Item Bias



n=8000; k=10
NB: green = item mean;

Results *(viii)*

NMAR (2) – Item Bias



Design:
Only even items affected by missing values

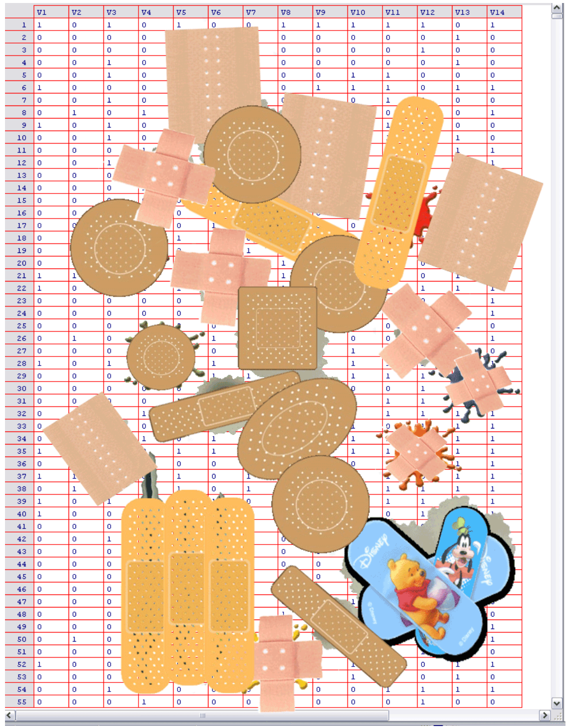
Conclusio (so far)

MCAR:
Largely unproblematic, all missing values handling methods performed equally well with respect to item bias. Random imputation and mean imputation outperformed the other principles (but structural missing/CML) in many instances.

LR-Test statistic appears to perform well, density misfit probably due to small number of replications (however, this requires affirmation).

NMAR:
If you have eRm (and perhaps someone who knows how to operate it) at hand, use CML and treat missing values as structurally missing.

If not, or if you deliberately want to impute, do not use fixed value imputation (e.g. setting missings to wrong answer). Rather, use item mean or just draw zeros and ones at random.



Thank You!

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