

Detecting Differential Item and Differential Step Functioning with Partial Credit Trees

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Differential Item Functioning (DIF)

is present when one or more items of a test

- ▶ are easier or harder to solve for certain subjects
- ▶ even though they have the same latent trait

Detecting DIF/DSF with PCMTrees

Testing for DIF in the RM

Standard tests
Model-based recursive partitioning

Extension to the PCM

DIF/DSF in the PCM
(Un)ordered threshold parameters

Visualization

Example: Verbal Aggression data

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Outline

Testing for DIF in the Rasch model

Standard model tests

Model-based recursive partitioning

Extending the model-based recursive partitioning approach to the Partial Credit Model (PCM)

Differential item and step functioning in the PCM

(Un)ordered threshold parameters in the PCM

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Standard model tests

- ▶ tests for k given groups
 - graphical test, Andersen's Likelihood-Ratio Test, Wald Tests
 - + straightforward interpretation
 - only detect DIF in specified groups
- ▶ latent-class approach
 - Rost's "Mixed" (mixture) Rasch model
 - + identifies previously unknown groups with DIF
 - groups are not directly interpretable
 - ⇒ 2nd step: describe groups with covariates (e.g., Cohen and Bolt, 2005)



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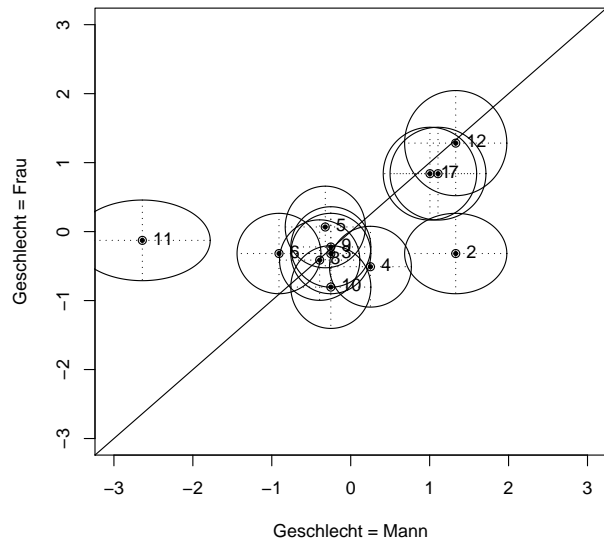
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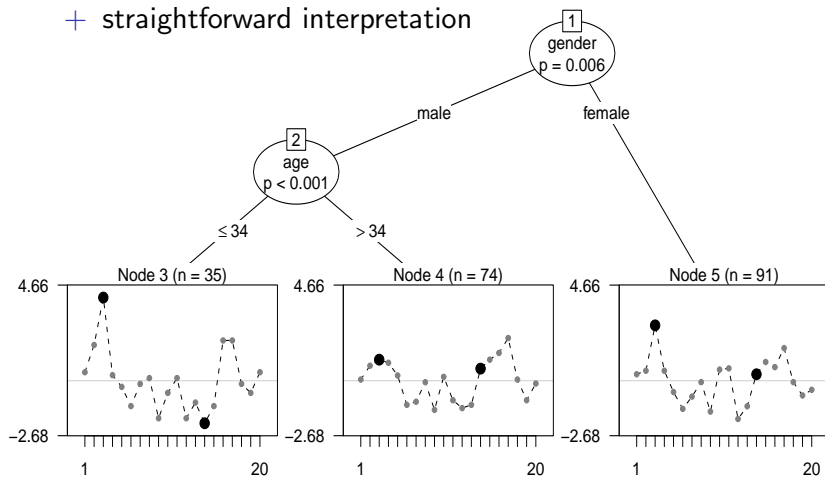


(Mair, Hatzinger, and Maier, 2010, package [eRm](#))



New: Model-based recursive partitioning

- + identifies previously unknown groups with DIF
- + straightforward interpretation



function [raschtree](#) in package [psychotree](#)



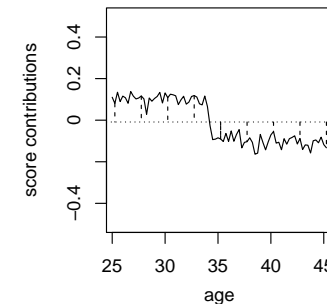
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Approach used in psychotree takes care of...

- ▶ selecting splitting variables \Leftrightarrow parameter instability tests



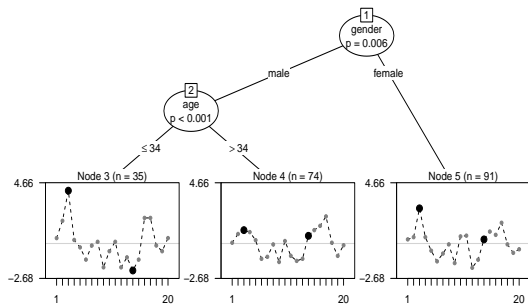
- ▶ selecting optimal cutpoints
- ▶ other multiple testing issues
 - ▶ between variables in each split
 - ▶ over successive splits

(Zeileis and Hornik, 2007; Zeileis, Hothorn, and Hornik, 2008; Strobl, Malley, and Tutz, 2009; Strobl, Kopf, and Zeileis, 2010a,b)



Extending the model-based partitioning approach

Rasch trees



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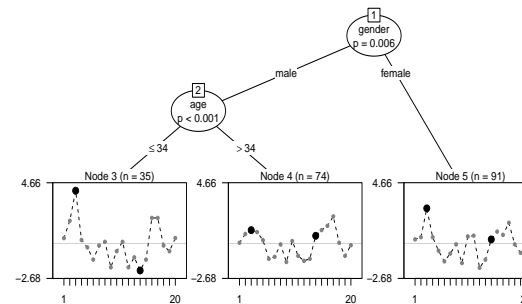
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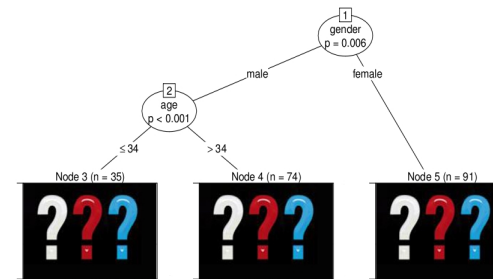
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Partial Credit trees



Extending the model-based partitioning approach

Rasch model

- ▶ scores are 0 or 1
- ▶ each item has one location parameter = difficulty
- ▶ DIF means item is more/less difficult for certain group

Partial Credit model

- ▶ scores are between 0 and m_j
- ▶ different parametrizations: e.g. m_j thresholds
- ▶ DIF means entire item is more/less difficult
- ▶ DSF means some steps are more/less difficult (may cancel out so there is no overall DIF)

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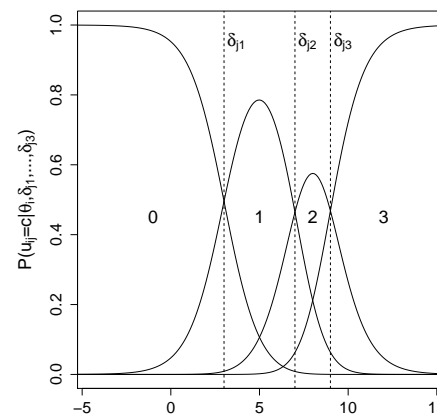
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(Un)ordered threshold parameters in the PCM



$$P(u_{ij} = c | \theta_i, \delta_{j1}, \dots, \delta_{jm_j}) = \frac{e^{\sum_{k=0}^c (\theta_i - \delta_{jk})}}{\sum_{l=0}^{m_j} e^{\sum_{k=0}^l (\theta_i - \delta_{jk})}}$$

with $\sum_{k=0}^0 (\theta_i - \delta_{jk}) = 0$

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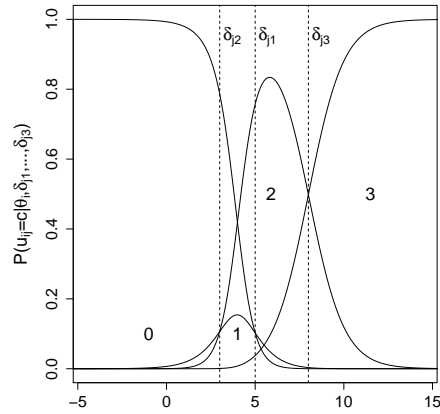
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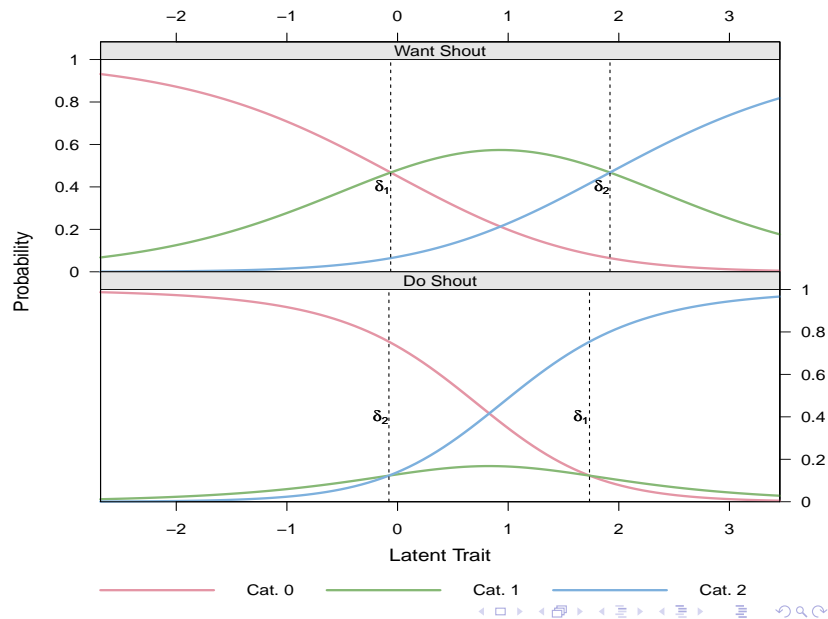
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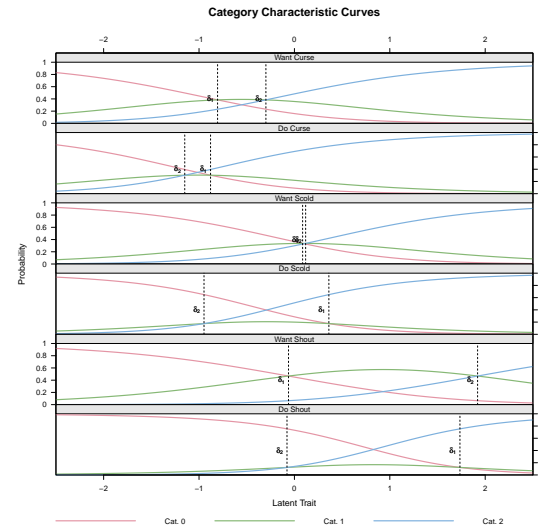


Visualization in Partial Credit trees

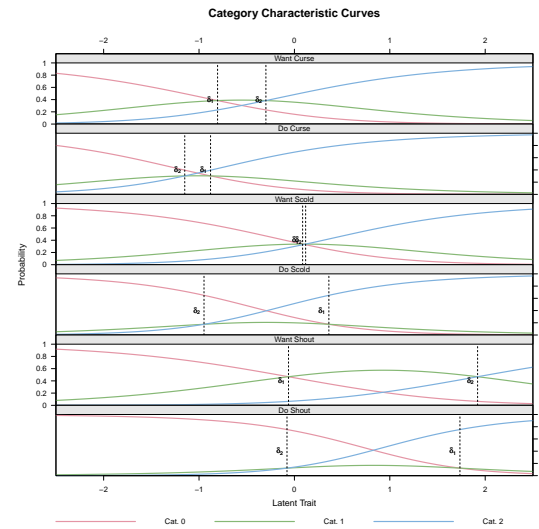
Category Characteristic Curves



Visualization in Partial Credit trees



Visualization in Partial Credit trees



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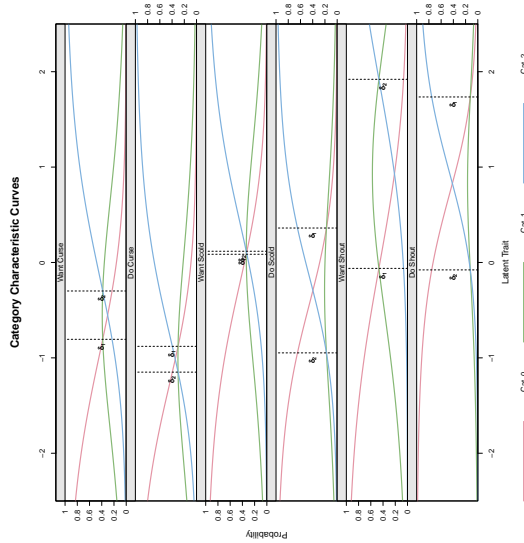
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Navigation icons: back, forward, search, etc.

Example: Verbal Aggression data

```
> data("VerbalAggression", package = "psychotools")
```

responses of 316 subjects to frustrating situations

- ▶ here: situation 4 (self-to-blame situation)
"The operator disconnects me when I used up my last 10 cents for a call."
- ▶ items: 3 verbally aggressive responses (curse, scold, shout) × 2 behavioural models (want, do)
- ▶ response categories: 0 = no, 1 = perhaps, 2 = yes
- ▶ covariates: gender, trait anger (assessed by the Dutch adaptation of the state-trait anger scale STAS)

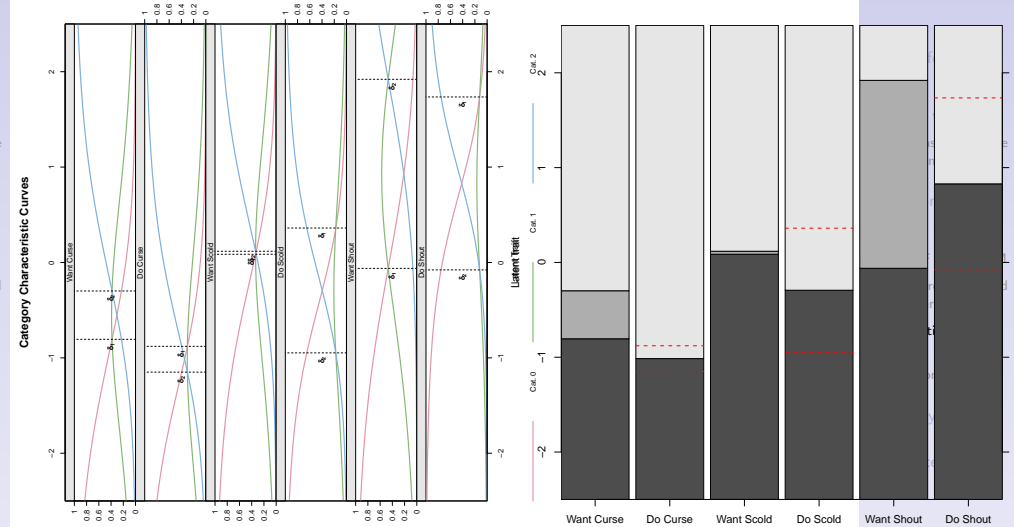
De Boeck and Wilson (2004), Smits, De Boeck, and Vansteelandt (2004), dichotomized version also available in package [difR](#) (Magis, Beland, and Raiche, 2011)

Navigation icons: back, forward, search, etc.

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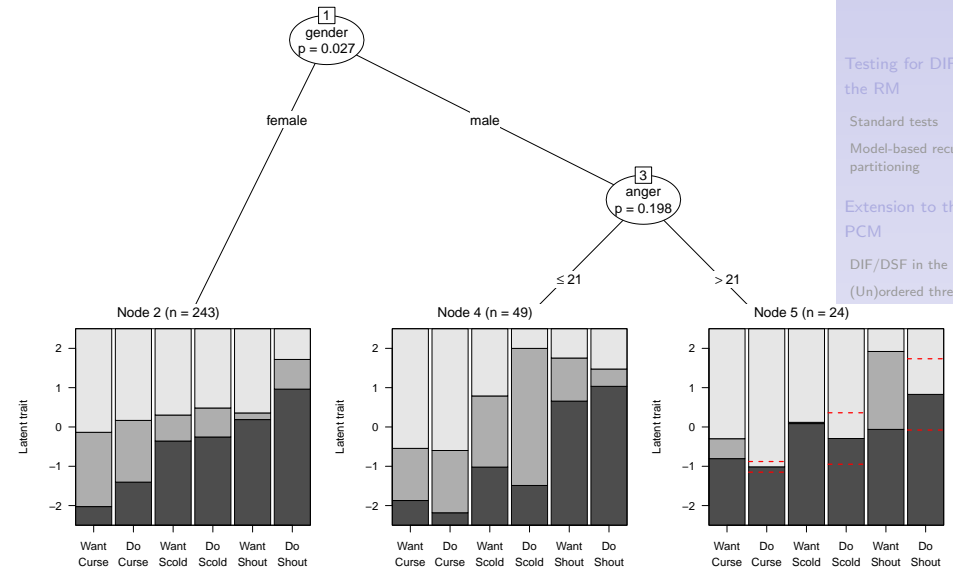
inspired by "effect plots" (Fox and Hong, 2009, package [effects](#))

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Partial Credit tree (tweaked a little for visualization)

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- ▶ can identify groups of subjects with DIF and DSF that
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 - ▶ are formed by (combinations of) observed covariates
 - ▶ with optimally selected cutpoints

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- ▶ available for
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 - ▶ and more to come

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- ▶ results are directly interpretable

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- ▶ available for
 - ▶ Rasch model
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 - ▶ and more to come
- ▶ results are directly interpretable, but keep in mind: observed covariates may be proxies for the true causes e.g.: gender \Leftrightarrow socialization, district \Leftrightarrow first language

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